Resume

We prove, that for each $r \in \mathbb{N}$, $n \in \mathbb{N}$ and $s \in \mathbb{N}$ there are a collection $\{y_i\}_{i=1}^{2s}$ of points $y_{2s} < y_{2s-1} < \cdots < y_1 < y_{2s} + 2\pi =: y_0$ and a 2π - periodic function $f \in C^{(\infty)}(\mathbb{R})$, such that

$$f''(t) \prod_{i=1}^{2s} (t - y_i) \geq 0, \quad t \in [y_{2s}, y_0], \tag{1}$$

and for each trigonometric polynomial T_n of degree $\leq n$ (of order $\leq 2n+1$), satisfying

$$T_n''(t) \prod_{i=1}^{2s} (t - y_i) \geq 0, \quad t \in [y_{2s}, y_0], \tag{2}$$

the inequality

$$n^{r-1} \|f - T_n\|_{C(\mathbb{R})} \geq c_r \|f^{(r)}\|_{C(\mathbb{R})}$$

holds, where $c_r > 0$ is a constant, depending only on r. Moreover, we prove, that for each $r = 0, 1, 2$ and any such collection $\{y_i\}_{i=1}^{2s}$ there is a 2π - periodic function $f \in C^{(r)}(\mathbb{R})$, such that $(-1)^{i-1} f$ is convex on $[y_i, y_{i-1}]$, $1 \leq i \leq 2s$, and, for each sequence $\{T_n\}_{n=0}^{\infty}$ of trigonometric polynomials T_n, satisfying (2), we have

$$\limsup_{n \to \infty} \frac{n^r \|f - T_n\|_{C(\mathbb{R})}}{\omega_4(f^{(r)}, 1/n)} = +\infty,$$

where ω_4 is the fourth modulus of continuity.

Furthermore, we consider the applications of comonotone and coconvex approximation in the theory of conflict dynamical systems. We construct a model of conflict dynamical system whose limit states are associated with singular distributions. It is proved that a criterion for the appearance of point spectrum in the limit distribution is the strategy with fixed priority. In all other cases, the limit distributions are pure singular continuous. The approximation approach simplifies the understanding of interaction process after a large number of steps have passed.